Discrete Dynamic Fracture with Finite Elements

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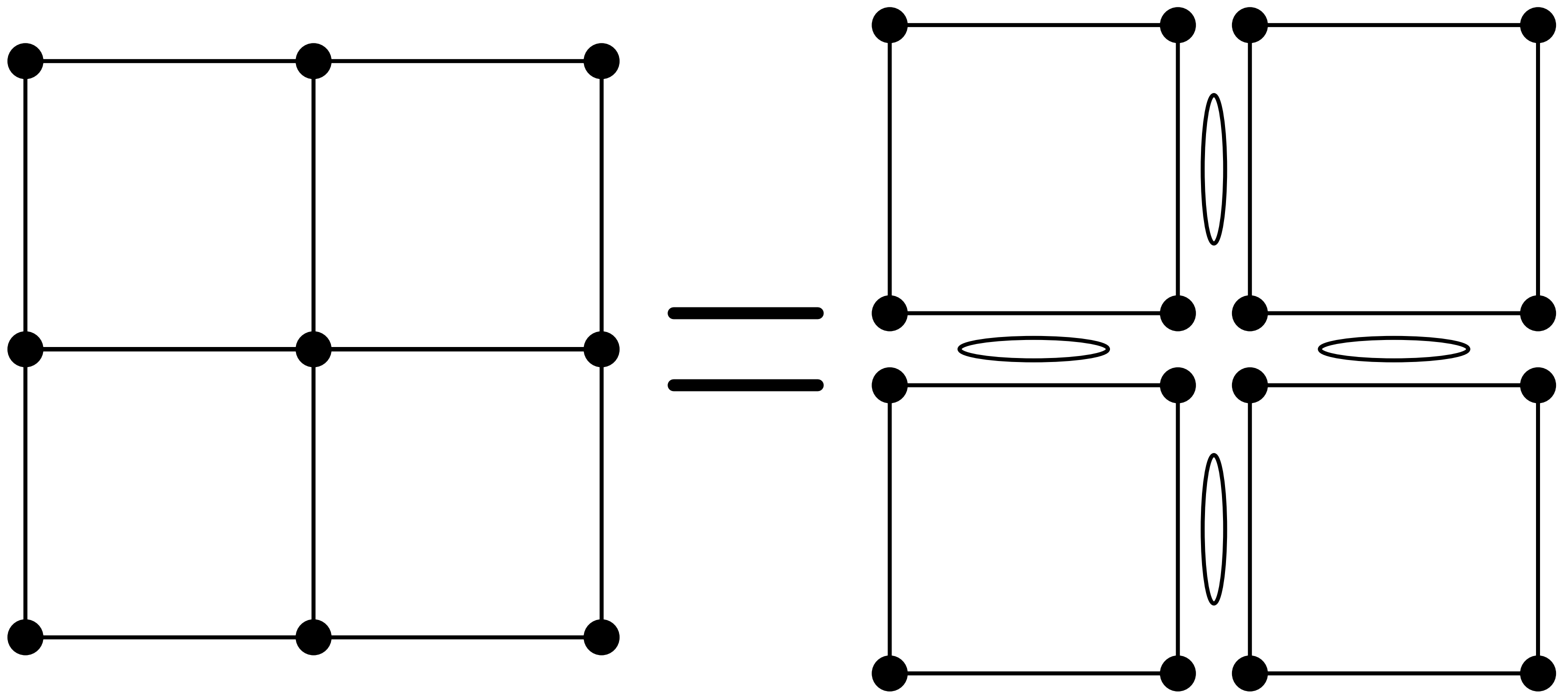
Discrete Dynamic Fracture

- Engineering Analysis Tool
  - Engineering Scale Structures
  - Predict Fracture of Mini Cracks
  - Geometry Changes/Surface Creation
  - Fragmentation
- Maintain “Standard” Modeling Approach
- Avoid
  - Deleting Elements
  - Remeshing
  - Damage Zones Representing Cracks

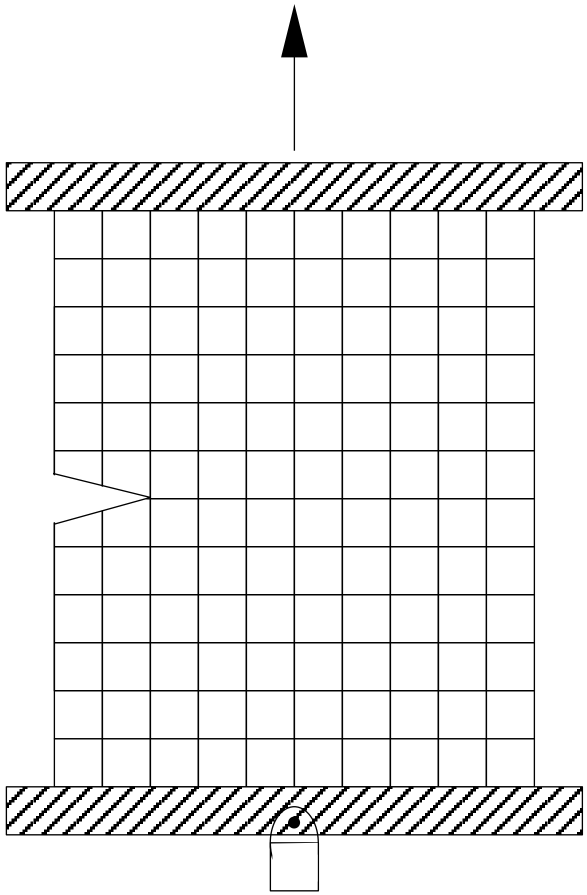
Approach

- Model a 2-D Structure w/a Distribution of Cracks
  - Maintain Displacement Continuity



Approach

- Define Failure for Virtual Cracks
- Discrete Fracture Follows Element Interfaces
- Path Otherwise not Predetermined
- Unique Nodal Connectivity
  - Maintain Original Mesh Definition



Development

- Hu - Washizu Energy Principle

$$\Pi_{HW}(\mathbf{u},\boldsymbol{\sigma},\boldsymbol{\varepsilon}) = \int_{\Omega}\left[\frac{1}{2}\varepsilon^{T}\mathbf{D}\varepsilon + \boldsymbol{\sigma}^{T}\left(\mathbf{L}\mathbf{u} - \varepsilon\right)\right]d\Omega - \Pi_{\mathrm{EXT}}$$

$$\Pi_{EXT} = \int_{\Omega}\left[\mathbf{u}^{T}\mathbf{b} + \boldsymbol{\sigma}^{T}\boldsymbol{\varepsilon}^{a}\right]d\Omega$$

- ε^{a} - externally applied strain field, due to small crack on element surface

Development

- Linear Approximation for 3 Fields (**u,σ,ε**)
- $\delta\Pi_{HW} = 0$
  - 3 Equations, 3 Unknowns

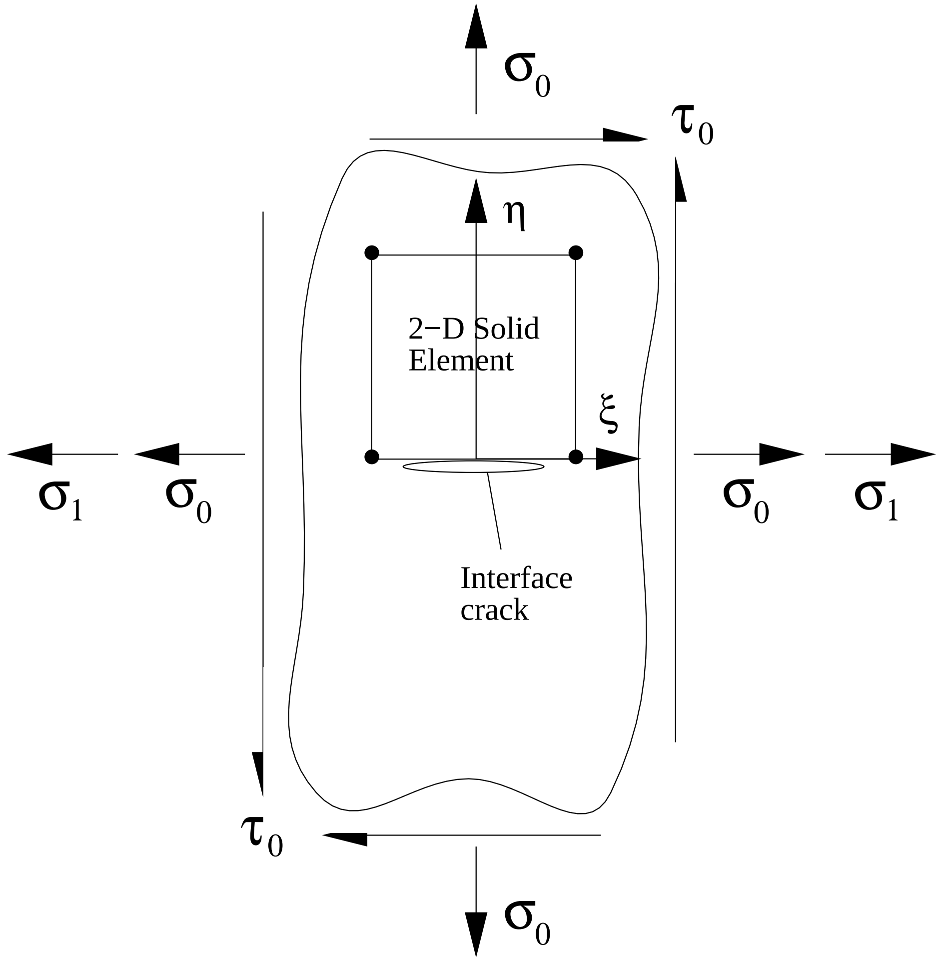
$$G^T A^{-T} H A^{-1} G d = f + G^T A^{-T} H A^{-1} Q$$

$$\mathbf{Q} \equiv \int_{\Omega} \mathbf{S}^T \varepsilon^a \, d\Omega$$

  - Fully Integrated/Numerically Integrated
- $\varepsilon^a \equiv 0$ - standard plane stress/strain finite element formulation

Development

- Applied Strain Field
  - Thru Crack in Infinite Elastic Plate
  - Stress - Westergaard Stress Function
  - Invert to get Strain Field



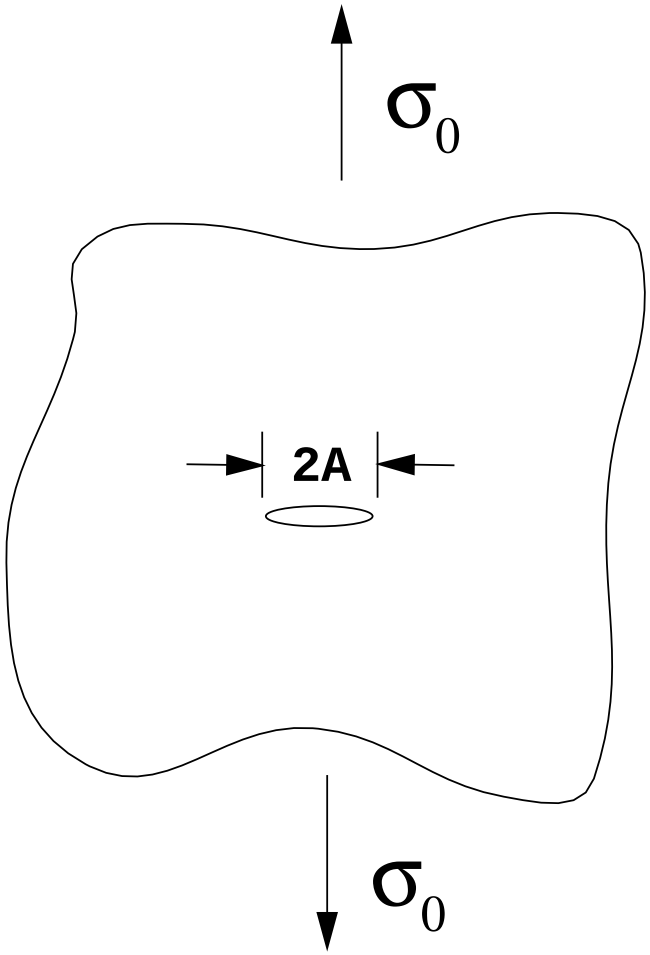
Development

- Applied Strain Field as a Function of Far Field Stress
  - Far Field Stresses ($\boldsymbol{\sigma_0}$, $\boldsymbol{\sigma_1}$)from Adjacent Elements
  - Integrate $\mathbf{Q} \equiv \int_{\Omega} \mathbf{S}^{T} \boldsymbol{\varepsilon}^{a} \, d\Omega$ numerically and add to standard load vector
  - Done for Each Crack on Surface - $Q = \Sigma Q_i$
  - Increases Compliance of Structure

Interface Failure

- Can we define failure criteria for the interface to match the macroscopic failure?
  - Only have local information
    - Stress from Adjacent Elements
    - Defined Crack Size

$$K_m = \sigma_0 \sqrt{\pi A}$$



Interface Failure

- Linear Elastic Fracture Mechanics
  - Apply macro equations to local problem

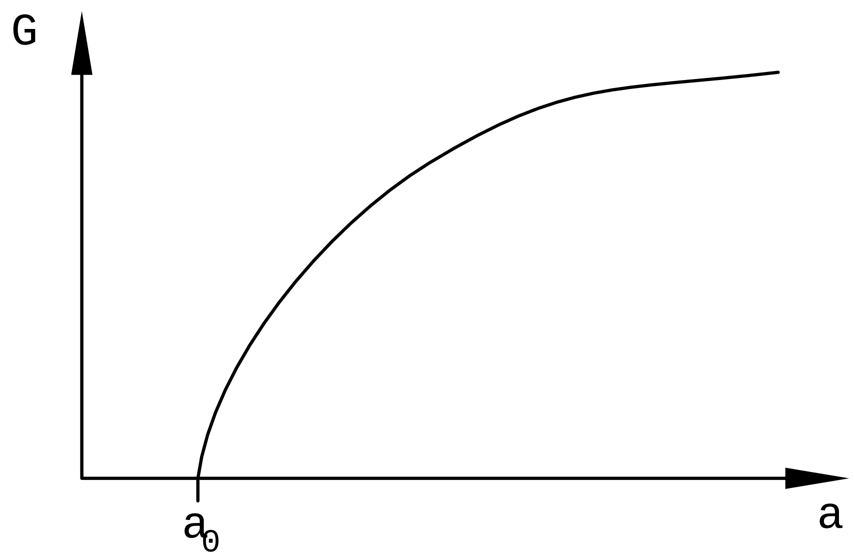
$$K_l = \sigma_l \sqrt{\pi a}$$

  - σ_l is higher at crack tip, *a* is smaller than macro crack
  - Failure Criterion: $K_l \geq K_{Ic}$
    - Instantaneous Growth
    - Strain Energy from Adjacent Elements
  - Problems with Growth Criteria

Interface Failure

- Elastic Plastic Fracture Mechanics
  - Assume Crack Growth Follows G-R Curve
  - Known for Material

$$G = \beta(\Delta a)^{\gamma} + \lambda$$



Interface Failure

- Find Strain Energy Release Rate

$$K = \sigma_l \sqrt{\pi \cdot a} \qquad G = \frac{K^2}{E}$$

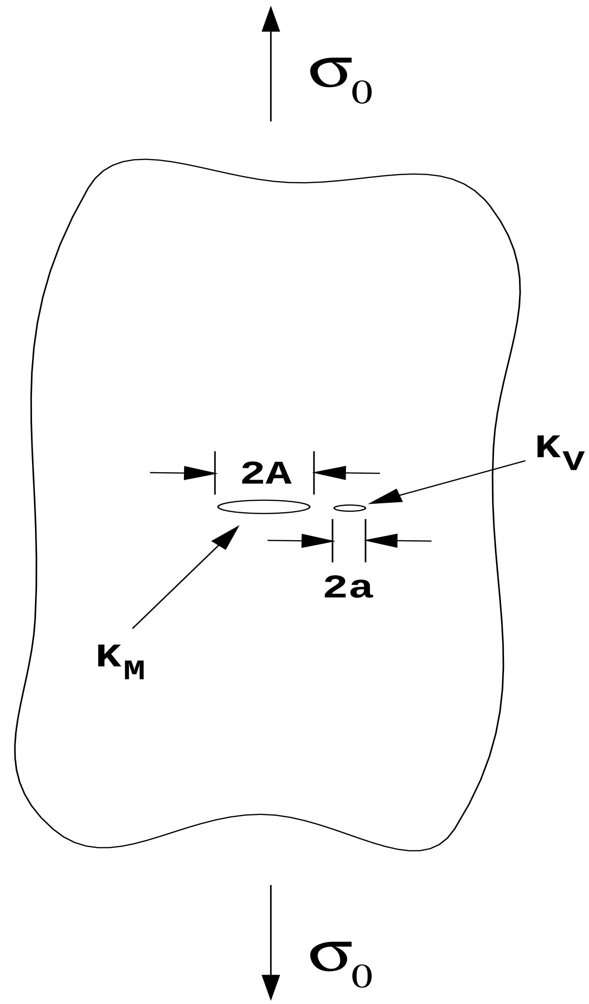
- Invert G(Δa) to get change in crack length

$$\Delta a = \left(\frac{G - \lambda}{\beta} \right)^{\frac{1}{\gamma}}$$

- Failure Criterion: *a* > interface width

Interface Failure

- Problem: balance between σ_l - *a* and σ_m - A incorrect
- Need better method of calculating stress intensity factor
- Look at a small crack in the vicinity of a large crack

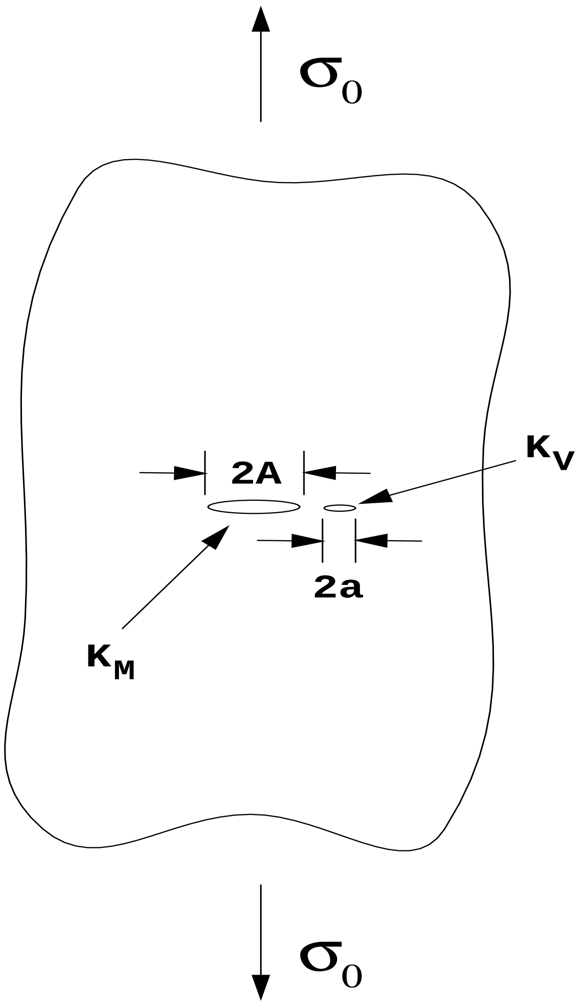


Interface Failure

- A>>a

$$\frac{K_M}{K_V} \approx \text{Constant}$$

- Get Macro Stress Intensity from Local State
- Use in EPFM
- Works Well for Large Straight Macro Cracks



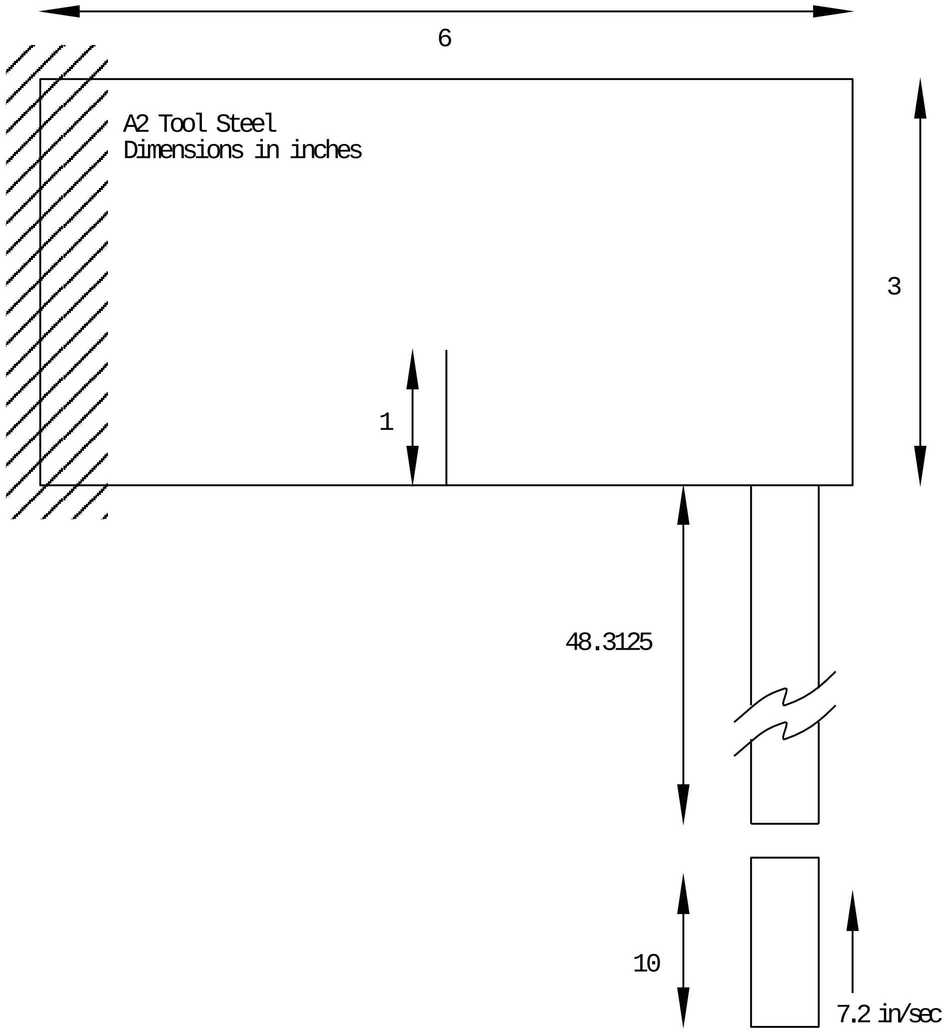
Interface Failure

- Still Working on Failure Criteria
- Fracture Paths Correct
- Load and Speed not Correct

Results

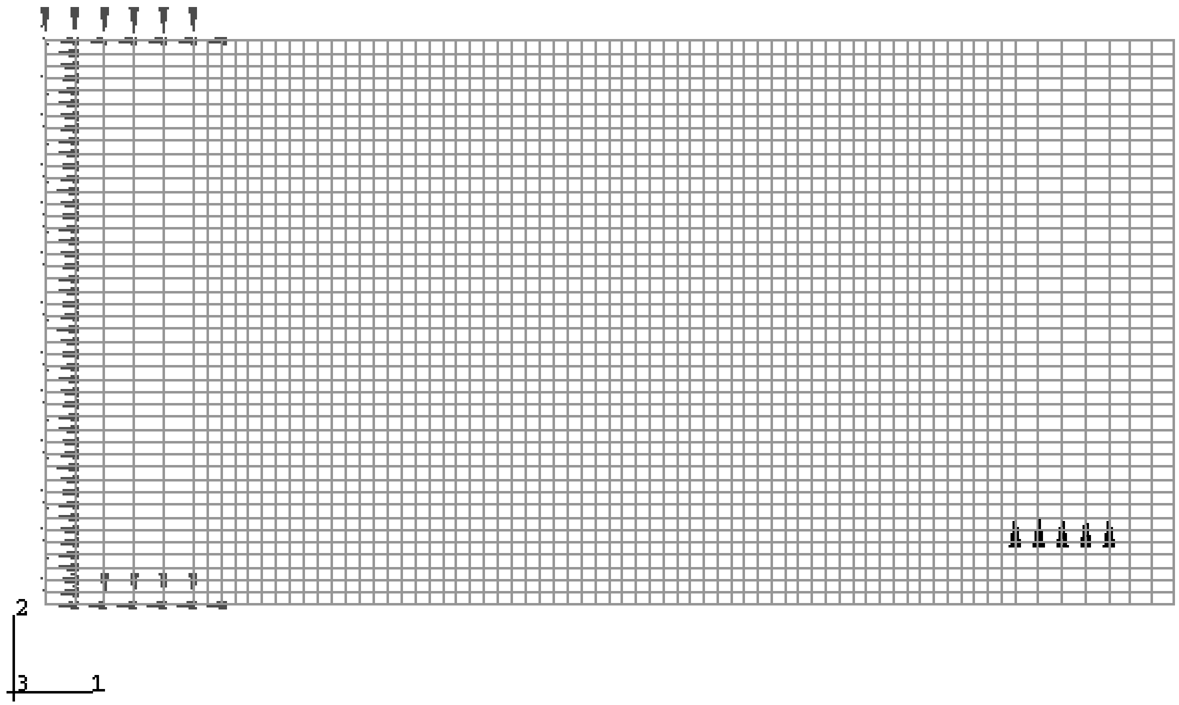
- Cantilever Impact
  - A2 Tool Steel

$$G = \left[16(\Delta a)^{\frac{3}{4}} + 10 \right] \left(\frac{in \cdot lb}{in^2} \right)$$

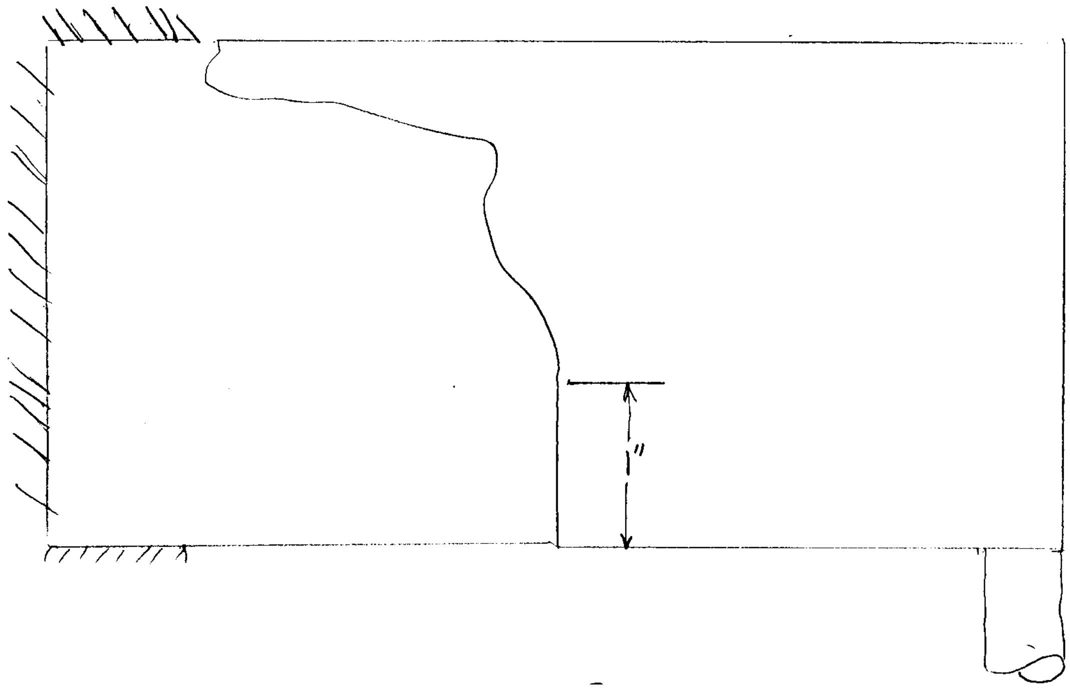


Results

- 3240 Plane Stress Elements -12960 Nodes



Results

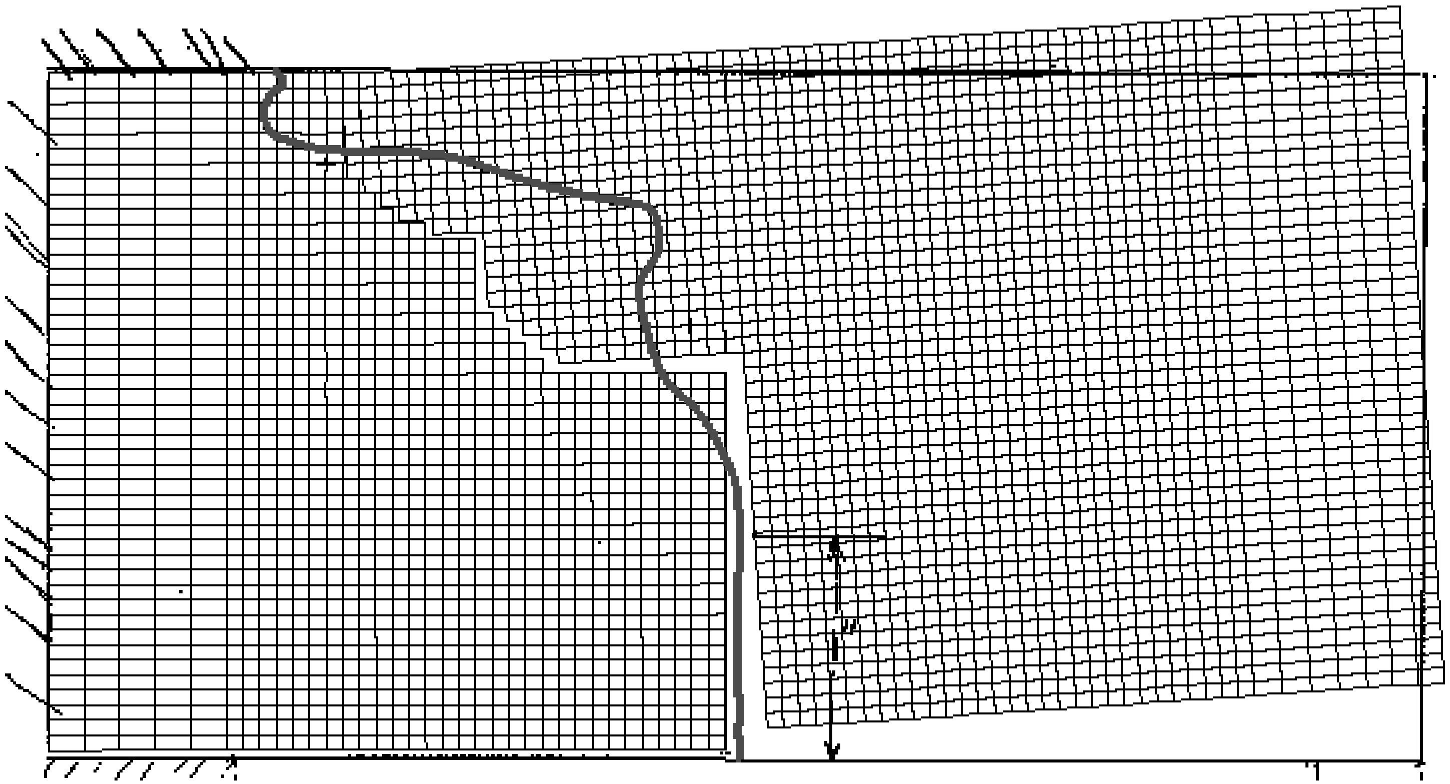


Tracing of Dynamic Fracture Sample

After Failure - A2 Steel

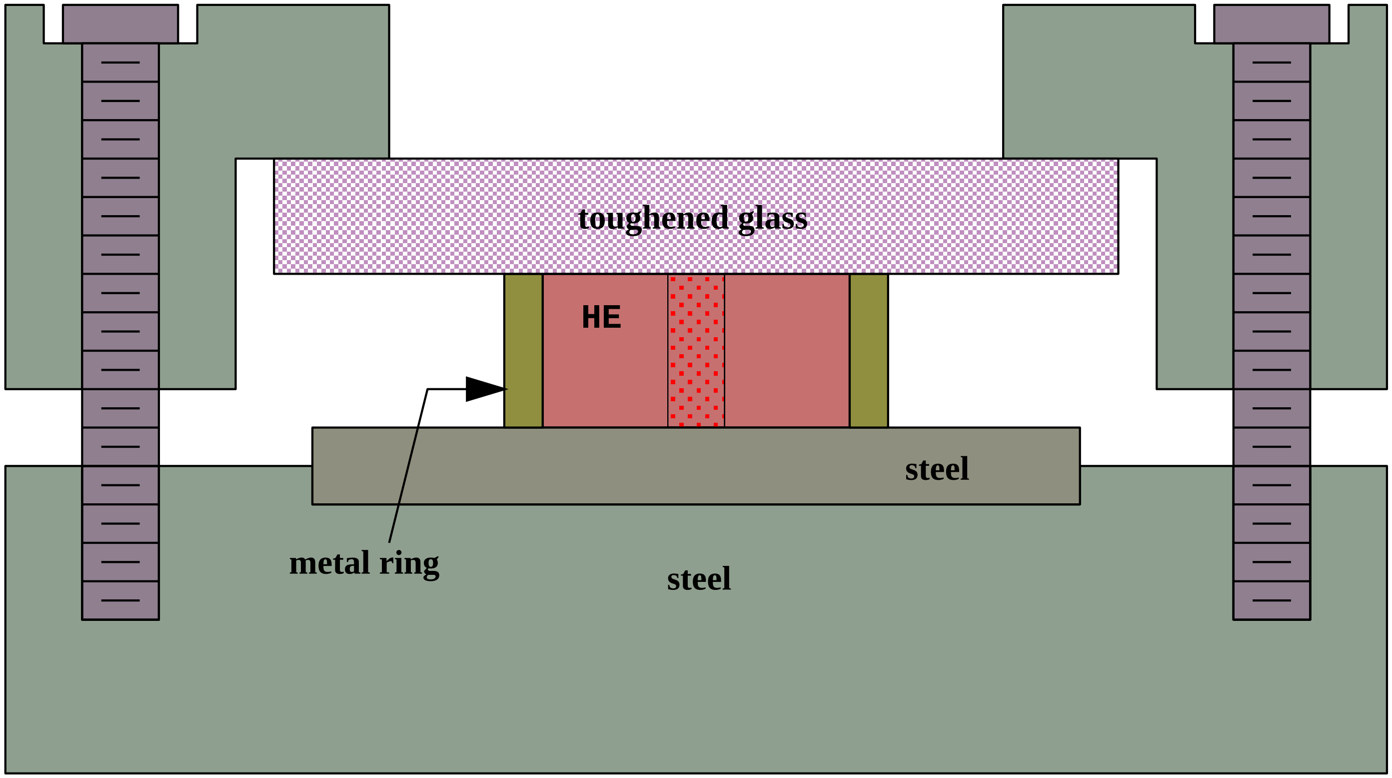
Liu, Stout, Gerken, Smith - 6/11/98

Results

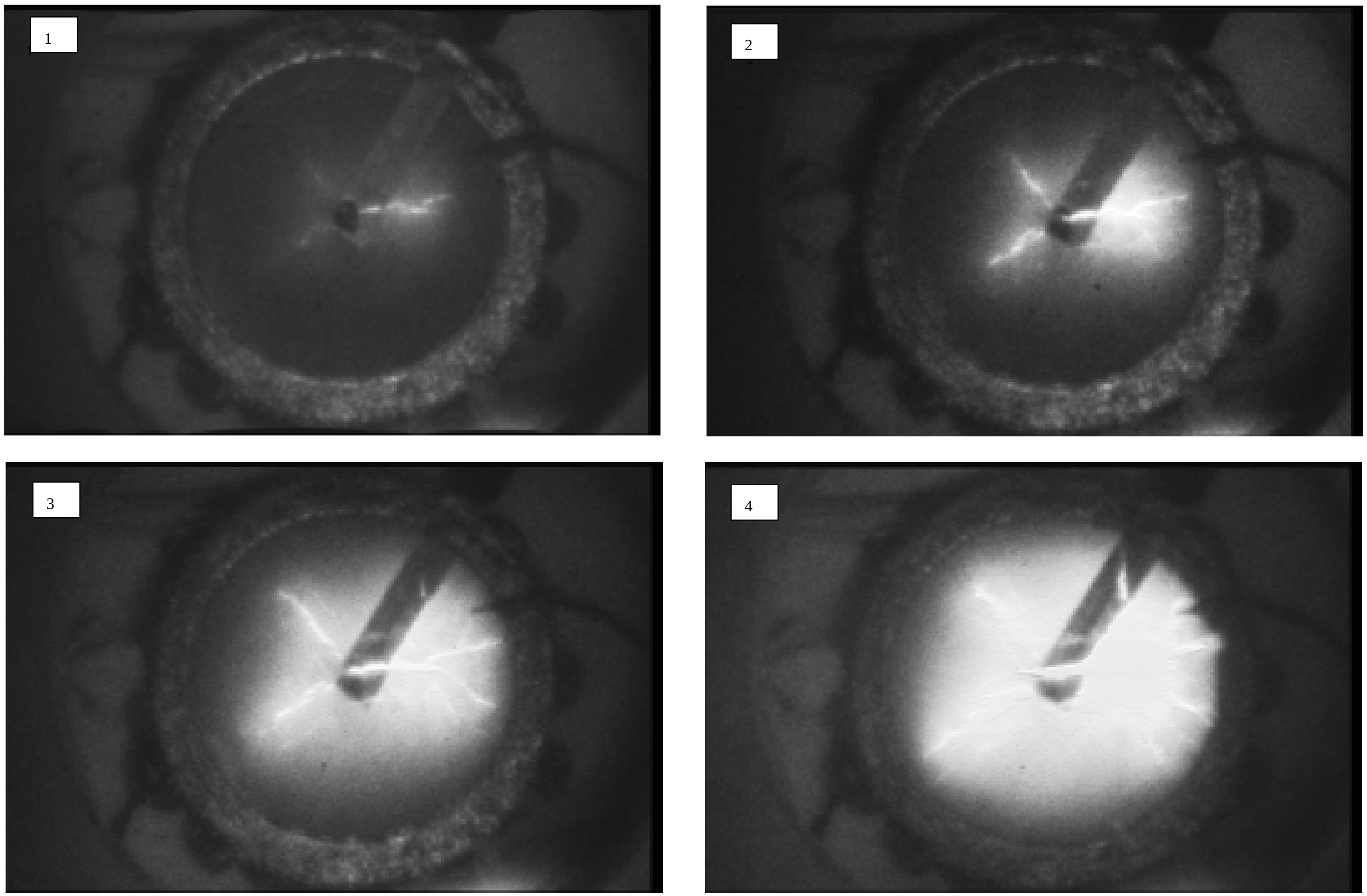


Results

- Mechanically Coupled Cook Off Experiment

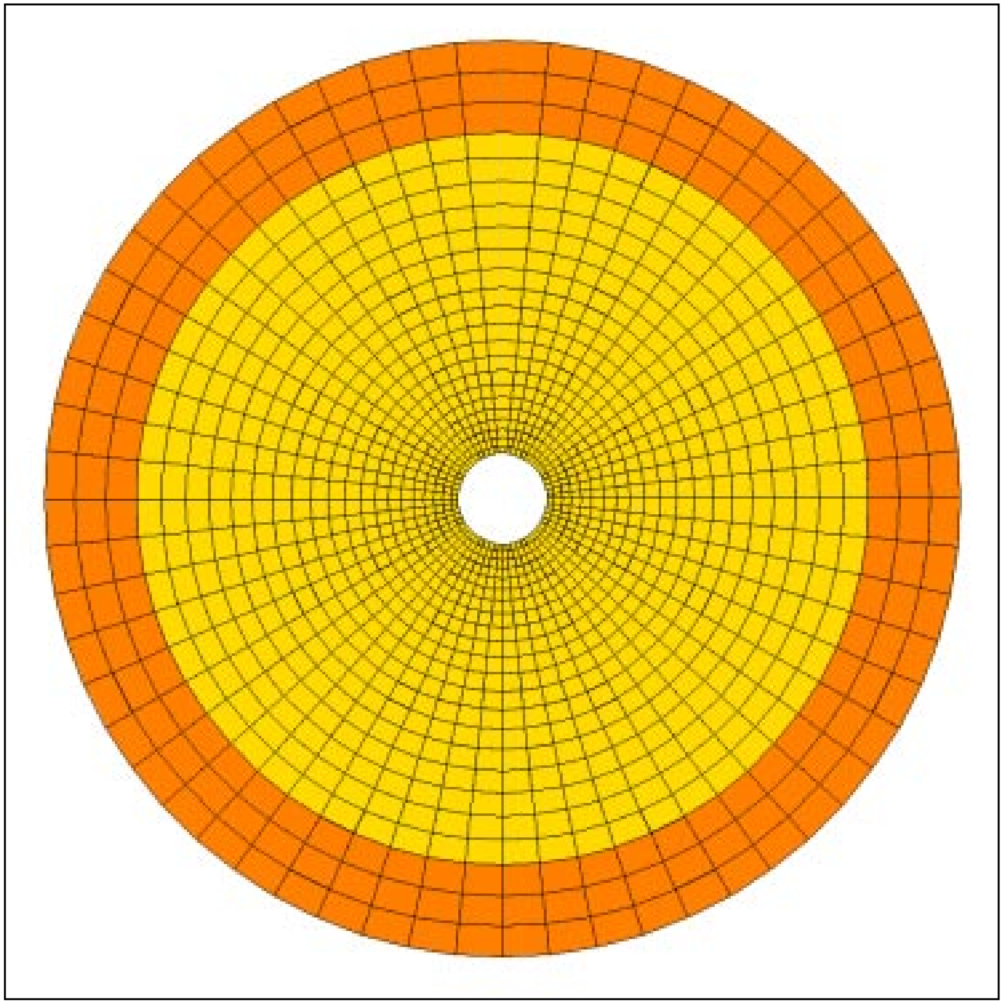


Results



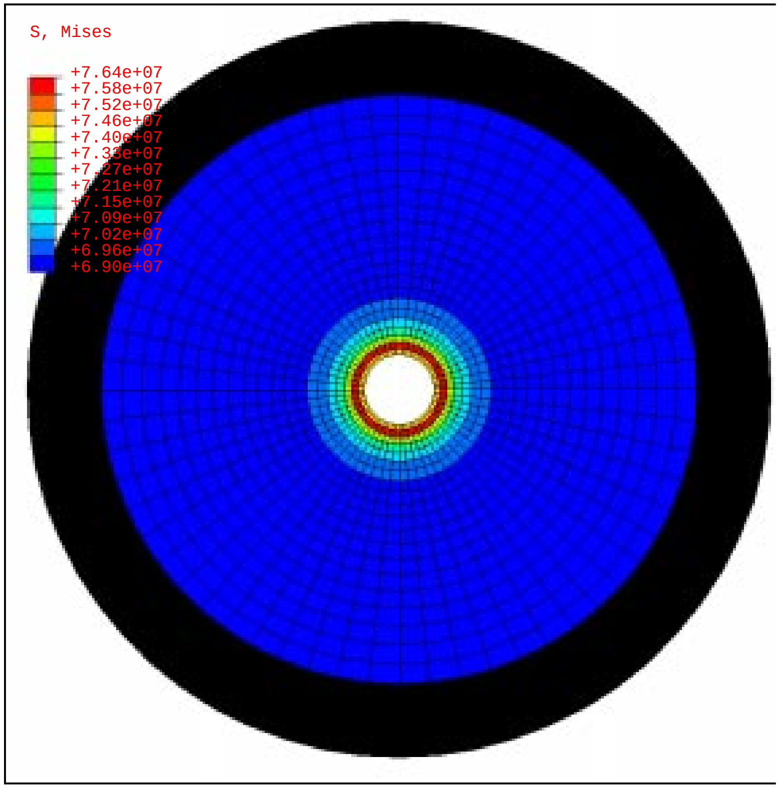
Results

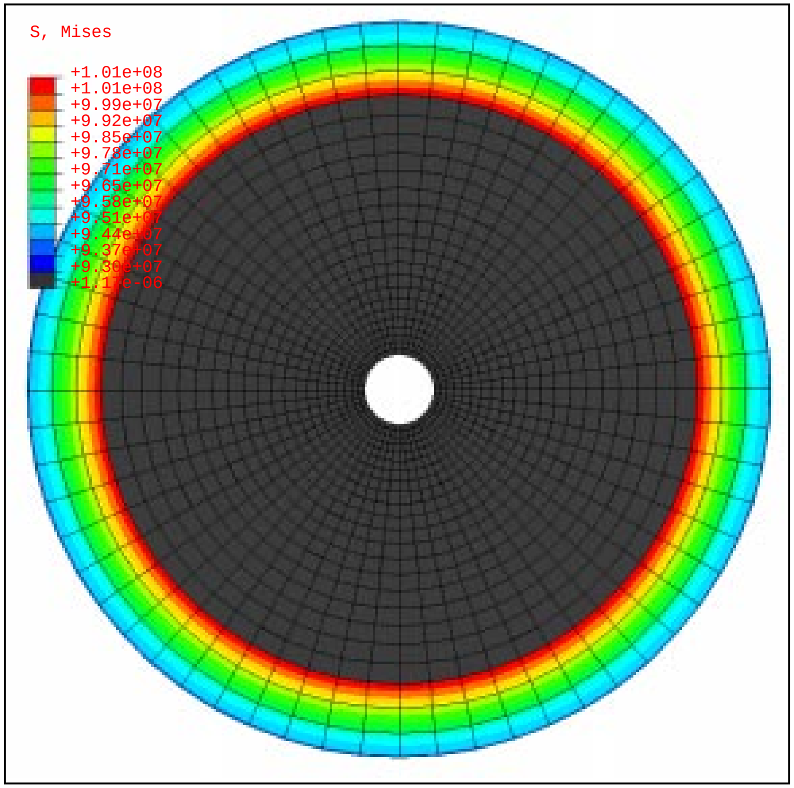
- Plane Strain
- Random Cracks
- Elastic/Plastic Cu
- ViscoSCRAM
- Thermal Expansion



Results

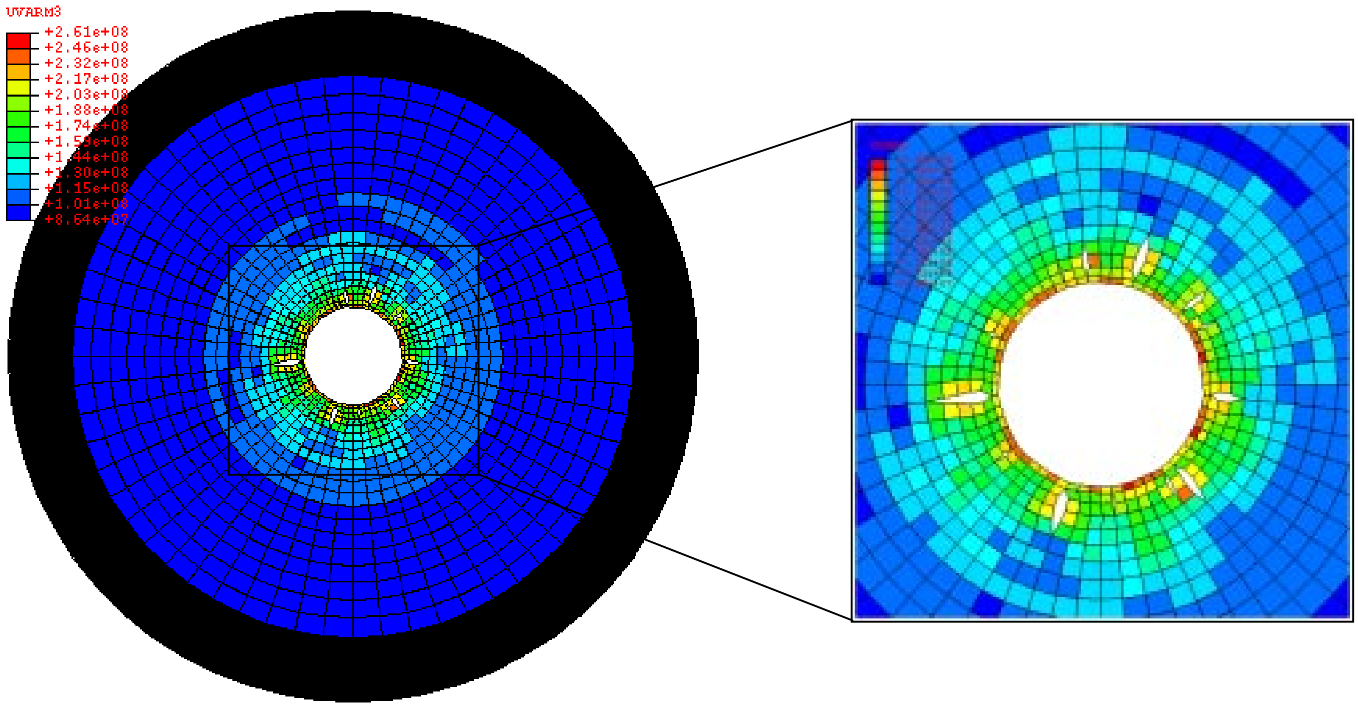
- Heat Up 120 K





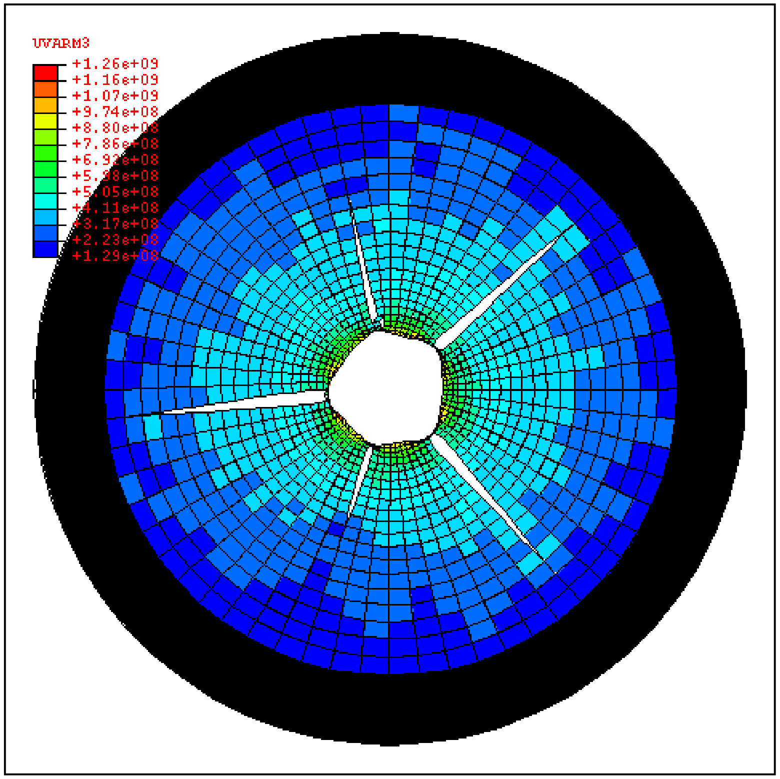
Results

- Apply Pressure - 5MPa/μsec



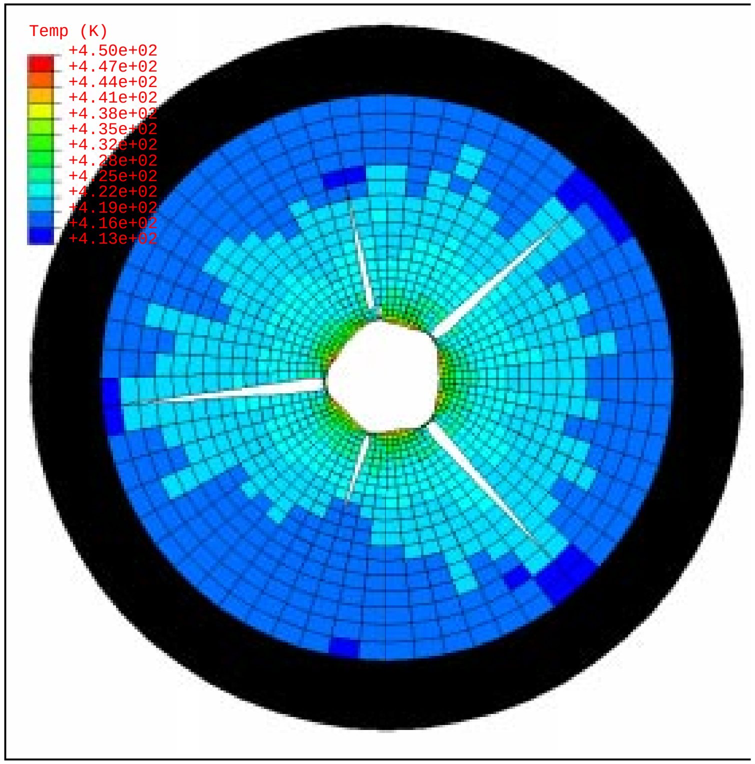
Results

- 3 to 5 Large Discrete Cracks Predicted

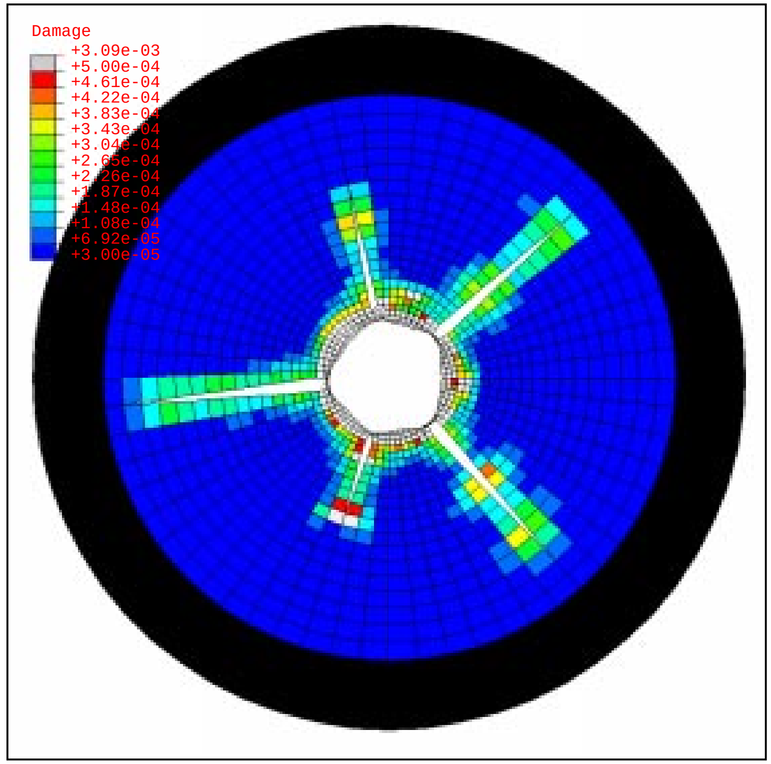


Results

- Cracks Increase Temp. and Damage



Temperature



Damage

Conclusions

- Discrete Fracture Model
  - Predicts Formation of Cracks
  - Predicts Appropriate Numbers of Cracks
  - Maintains “Standard” Modeling Approach
  - Failure Criteria Need Improvement
- Reproducing Experimental Results
  - Crack Paths Look Good